

This Page Is Inserted by IFW Operations  
and is not a part of the Official Record

## **BEST AVAILABLE IMAGES**

Defective images within this document are accurate representations of the original documents submitted by the applicant.

Defects in the images may include (but are not limited to):

- BLACK BORDERS
- TEXT CUT OFF AT TOP, BOTTOM OR SIDES
- FADED TEXT
- ILLEGIBLE TEXT
- SKEWED/SLANTED IMAGES
- COLORED PHOTOS
- BLACK OR VERY BLACK AND WHITE DARK PHOTOS
- GRAY SCALE DOCUMENTS

**IMAGES ARE BEST AVAILABLE COPY.**

**As rescanning documents *will not* correct images,  
please do not report the images to the  
Image Problems Mailbox.**

SIMO MÄENPÄÄ  
P. 2449425

Second Edition

# Field and Wave Electromagnetics

**David K. Cheng**

Life Fellow, IEEE;  
Fellow, I.E.E.; C. Eng.



ADDISON-WESLEY PUBLISHING COMPANY

Reading, Massachusetts • Menlo Park, California • New York  
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn  
Sydney • Singapore • Tokyo • Madrid • San Juan

**WORLD STUDENT SERIES EDITION**

**This book is in the Addison-Wesley Series in Electrical Engineering**

**Barbara Riskind: Sponsoring Editor**  
**Karen Myer: Production Supervisor**  
**Hugh Crawford: Manufacturing Supervisor**  
**Joseph K. Vetere: Technical Art Consultant**  
**Catherine Dorin: Interior Designer**  
**Marshall Henrichs: Cover Designer**  
**Perry DuMoulin: Production Coordinator**

**Copyright © 1989 by Addison-Wesley Publishing Company, Inc.**

**All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America. Published simultaneously in Canada.**

**ISBN 0-201-51820-1**

**45678-DO-939291**

**Fourth Printing, January 1991.**

## Preface

The many books on introductory electromagnetics can be roughly divided into two main groups. The first group takes the traditional development: starting with the experimental laws, generalizing them in steps, and finally synthesizing them in the form of Maxwell's equations. This is an inductive approach. The second group takes the axiomatic development: starting with Maxwell's equations, identifying each with the appropriate experimental law, and specializing the general equations to static and time-varying situations for analysis. This is a deductive approach. A few books begin with a treatment of the special theory of relativity and develop all of electromagnetic theory from Coulomb's law of force; but this approach requires the discussion and understanding of the special theory of relativity first and is perhaps best suited for a course at an advanced level.

Proponents of the traditional development argue that it is the way electromagnetic theory was unraveled historically (from special experimental laws to Maxwell's equations), and that it is easier for the students to follow than the other methods. I feel, however, that the way a body of knowledge was unraveled is not necessarily the best way to teach the subject to students. The topics tend to be fragmented and cannot take full advantage of the conciseness of vector calculus. Students are puzzled at, and often form a mental block to, the subsequent introduction of gradient, divergence, and curl operations. As a process for formulating an electromagnetic model, this approach lacks cohesiveness and elegance.

The axiomatic development usually begins with the set of four Maxwell's equations, either in differential or in integral form, as fundamental postulates. These are equations of considerable complexity and are difficult to master. They are likely to cause consternation and resistance in students who are hit with all of them at the beginning of a book. Alert students will wonder about the meaning of the field vectors and about the necessity and sufficiency of these general equations. At the initial stage students tend to be confused about the concepts of the electromagnetic model, and they are not yet comfortable with the associated mathematical manipulations. In any case, the general Maxwell's equations are soon simplified to apply to static fields.

---

## Contents

<b>1</b>	<b>The Electromagnetic Model</b>	<b>1</b>
1-1	Introduction	1
1-2	The Electromagnetic Model	3
1-3	SI Units and Universal Constants	8
	Review Questions	10
<b>2</b>	<b>Vector Analysis</b>	<b>11</b>
2-1	Introduction	11
2-2	Vector Addition and Subtraction	12
2-3	Products of Vectors	14
	2-3.1 Scalar or Dot Product	14
	2-3.2 Vector or Cross Product	16
	2-3.3 Product of Three Vectors	18
2-4	Orthogonal Coordinate Systems	20
	2-4.1 Cartesian Coordinates	23
	2-4.2 Cylindrical Coordinates	27
	2-4.3 Spherical Coordinates	31
2-5	Integrals Containing Vector Functions	37
2-6	Gradient of a Scalar Field	42
2-7	Divergence of a Vector Field	46
2-8	Divergence Theorem	50
2-9	Curl of a Vector Field	54
2-10	Stokes's Theorem	58
		ix

## Contents

2-11	Two Null Identities	61
2-11.1	Identity I	61
2-11.2	Identity II	62
2-12	Helmholtz's Theorem	63
	Review Questions	66
	Problems	67

### 3 Static Electric Fields 72

3-1	Introduction	72
3-2	Fundamental Postulates of Electrostatics in Free Space	74
3-3	Coulomb's Law	77
3-3.1	Electric Field Due to a System of Discrete Charges	82
3-3.2	Electric Field Due to a Continuous Distribution of Charge	84
3-4	Gauss's Law and Applications	87
3-5	Electric Potential	92
3-5.1	Electric Potential Due to a Charge Distribution	94
3-6	Conductors in Static Electric Field	100
3-7	Dielectrics in Static Electric Field	105
3-7.1	Equivalent Charge Distributions of Polarized Dielectrics	106
3-8	Electric Flux Density and Dielectric Constant	109
3-8.1	Dielectric Strength	114
3-9	Boundary Conditions for Electrostatic Fields	116
3-10	Capacitance and Capacitors	121
3-10.1	Series and Parallel Connections of Capacitors	126
3-10.2	Capacitances in Multiconductor Systems	129
3-10.3	Electrostatic Shielding	132
3-11	Electrostatic Energy and Forces	133
3-11.1	Electrostatic Energy in Terms of Field Quantities	137
3-11.2	Electrostatic Forces	140
	Review Questions	143
	Problems	145

### 4 Solution of Electrostatic Problems 152

4-1	Introduction	152
4-2	Poisson's and Laplace's Equations	152
4-3	Uniqueness of Electrostatic Solutions	157

## Contents

xi

4-4	Method of Images	159
4-4.1	Point Charge and Conducting Planes	161
4-4.2	Line Charge and Parallel Conducting Cylinder	162
4-4.3	Point Charge and Conducting Sphere	170
4-4.4	Charged Sphere and Grounded Plane	172
4-5	Boundary-Value Problems in Cartesian Coordinates	174
4-6	Boundary-Value Problems in Cylindrical Coordinates	183
4-7	Boundary-Value Problems in Spherical Coordinates	188
	Review Questions	192
	Problems	193

## 5 Steady Electric Currents 198

5-1	Introduction	198
5-2	Current Density and Ohm's Law	199
5-3	Electromotive Force and Kirchhoff's Voltage Law	205
5-4	Equation of Continuity and Kirchhoff's Current Law	208
5-5	Power Dissipation and Joule's Law	210
5-6	Boundary Conditions for Current Density	211
5-7	Resistance Calculations	215
	Review Questions	219
	Problems	220

## 6 Static Magnetic Fields 225

6-1	Introduction	225
6-2	Fundamental Postulates of Magnetostatics in Free Space	226
6-3	Vector Magnetic Potential	232
6-4	The Biot-Savart Law and Applications	234
6-5	The Magnetic Dipole	239
6-5.1	Scalar Magnetic Potential	242
6-6	Magnetization and Equivalent Current Densities	243
6-6.1	Equivalent Magnetization Charge Densities	247
6-7	Magnetic Field Intensity and Relative Permeability	249
6-8	Magnetic Circuits	251
6-9	Behavior of Magnetic Materials	257
6-10	Boundary Conditions for Magnetostatic Fields	262
6-11	Inductances and Inductors	266

xii

## Contents

6-12	Magnetic Energy	
6-12.1	Magnetic Energy in Terms of Field Quantities	279
6-13	Magnetic Forces and Torques	
6-13.1	Hall Effect	282
6-13.2	Forces and Torques on Current-Carrying Conductors	283
6-13.3	Forces and Torques in Terms of Stored Magnetic Energy	289
6-13.4	Forces and Torques in Terms of Mutual Inductance	292
	Review Questions	294
	Problems	296

## 7 Time-Varying Fields and Maxwell's Equations 307

7-1	Introduction	307
7-2	Faraday's Law of Electromagnetic Induction	308
7-2.1	A Stationary Circuit in a Time-Varying Magnetic Field	309
7-2.2	Transformers	310
7-2.3	A Moving Conductor in a Static Magnetic Field	314
7-2.4	A Moving Circuit in a Time-Varying Magnetic Field	317
7-3	Maxwell's Equations	
7-3.1	Integral Form of Maxwell's Equations	323
7-4	Potential Functions	326
7-5	Electromagnetic Boundary Conditions	329
7-5.1	Interface between Two Lossless Linear Media	330
7-5.2	Interface between a Dielectric and a Perfect Conductor	331
7-6	Wave Equations and Their Solutions	332
7-6.1	Solution of Wave Equations for Potentials	333
7-6.2	Source-Free Wave Equations	334
7-7	Time-Harmonic Fields	
7-7.1	The Use of Phasors—A Review	336
7-7.2	Time-Harmonic Electromagnetics	338
7-7.3	Source-Free Fields in Simple Media	340
7-7.4	The Electromagnetic Spectrum	343
	Review Questions	346
	Problems	347

## 8 Plane Electromagnetic Waves 354

8-1	Introduction	
8-2	Plane Waves in Lossless Media	354
8-2.1	Doppler Effect	355



## Contents

8-2.2	Transverse Electromagnetic Waves	361
8-2.3	Polarization of Plane Waves	364
8-3	Plane Waves in Lossy Media	367
8-3.1	Low-Loss Dielectrics	368
8-3.2	Good Conductors	369
8-3.3	Ionized Gases	373
8-4	Group Velocity	375
8-5	Flow of Electromagnetic Power and the Poynting Vector	379
8-5.1	Instantaneous and Average Power Densities	382
8-6	Normal Incidence at a Plane Conducting Boundary	386
8-7	Oblique Incidence at a Plane Conducting Boundary	390
8-7.1	Perpendicular Polarization	390
8-7.2	Parallel Polarization	395
8-8	Normal Incidence at a Plane Dielectric Boundary	397
8-9	Normal Incidence at Multiple Dielectric Interfaces	401
8-9.1	Wave Impedance of the Total Field	403
8-9.2	Impedance Transformation with Multiple Dielectrics	404
8-10	Oblique Incidence at a Plane Dielectric Boundary	406
8-10.1	Total Reflection	408
8-10.2	Perpendicular Polarization	411
8-10.3	Parallel Polarization	414
	Review Questions	417
	Problems	419

## 9

## Theory and Applications of Transmission Lines 427

9-1	Introduction	427
9-2	Transverse Electromagnetic Wave along a Parallel-Plate Transmission Line	429
9-2.1	Lossy Parallel-Plate Transmission Lines	433
9-2.2	Microstrip Lines	435
9-3	General Transmission-Line Equations	437
9-3.1	Wave Characteristics on an Infinite Transmission Line	439
9-3.2	Transmission-Line Parameters	444
9-3.3	Attenuation Constant from Power Relations	447
9-4	Wave Characteristics on Finite Transmission Lines	449
9-4.1	Transmission Lines as Circuit Elements	454
9-4.2	Lines with Resistive Termination	460
9-4.3	Lines with Arbitrary Termination	465
9-4.4	Transmission-Line Circuits	467
9-5	Transients on Transmission Lines	471
9-5.1	Reflection Diagrams	474

xiv

## Contents

9-5.2	Pulse Excitation 478	
9-5.3	Initially Charged Line 480	
9-5.4	Line with Reactive Load 482	
9-6	The Smith Chart	
9-6.1	Smith-Chart Calculations for Lossy Lines 495	485
9-7	Transmission-Line Impedance Matching	
9-7.1	Impedance Matching by Quarter-Wave Transformer 497	497
9-7.2	Single-Stub Matching 501	
9-7.3	Double-Stub Matching 505	
	Review Questions	509
	Problems	512

## 10 Waveguides and Cavity Resonators 520

10-1	Introduction	520
10-2	General Wave Behaviors along Uniform Guiding Structures	521
10-2.1	Transverse Electromagnetic Waves 524	
10-2.2	Transverse Magnetic Waves 525	
10-2.3	Transverse Electric Waves 529	
10-3	Parallel-Plate Waveguide	534
10-3.1	TM Waves between Parallel Plates 534	
10-3.2	TE Waves between Parallel Plates 539	
10-3.3	Energy-Transport Velocity 541	
10-3.4	Attenuation in Parallel-Plate Waveguides 543	
10-4	Rectangular Waveguides	547
10-4.1	TM Waves in Rectangular Waveguides 547	
10-4.2	TE Waves in Rectangular Waveguides 551	
10-4.3	Attenuation in Rectangular Waveguides 555	
10-4.4	Discontinuities in Rectangular Waveguides 559	
10-5	Circular Waveguides	562
10-5.1	Bessel's Differential Equation and Bessel Functions 563	
10-5.2	TM Waves in Circular Waveguides 567	
10-5.3	TE Waves in Circular Waveguides 569	
10-6	Dielectric Waveguides	572
10-6.1	TM Waves along a Dielectric Slab 572	
10-6.2	TE Waves along a Dielectric Slab 576	
10-6.3	Additional Comments on Dielectric Waveguides 579	
10-7	Cavity Resonators	582
10-7.1	Rectangular Cavity Resonators 582	
10-7.2	Quality Factor of Cavity Resonator 586	
10-7.3	Circular Cavity Resonator 589	
	Review Questions	592
	Problems	594

## Contents

xv

**11 Antennas and Radiating Systems 600**

11-1	Introduction	600
11-2	Radiation Fields of Elemental Dipoles	602
11-2.1	The Elemental Electric Dipole	602
11-2.2	The Elemental Magnetic Dipole	605
11-3	Antenna Patterns and Antenna Parameters	607
11-4	Thin Linear Antennas	614
11-4.1	The Half-Wave Dipole	617
11-4.2	Effective Antenna Length	619
11-5	Antenna Arrays	621
11-5.1	Two-Element Arrays	622
11-5.2	General Uniform Linear Arrays	625
11-6	Receiving Antennas	631
11-6.1	Internal Impedance and Directional Pattern	632
11-6.2	Effective Area	634
11-6.3	Backscatter Cross Section	637
11-7	Transmit-Receive Systems	639
11-7.1	Friis Transmission Formula and Radar Equation	639
11-7.2	Wave Propagation near Earth's Surface	642
11-8	Some Other Antenna Types	643
11-8.1	Traveling-Wave Antennas	643
11-8.2	Helical Antennas	645
11-8.3	Yagi-Uda Antenna	648
11-8.4	Broadband Antennas	650
11-9	Aperture Radiators	655
	References	661
	Review Questions	662
	Problems	664

**Appendixes****A Symbols and Units 671**

A-1	Fundamental SI (Rationalized MKSA) Units	671
A-2	Derived Quantities	671
A-3	Multiples and Submultiples of Units	673

**B Some Useful Material Constants 674**

B-1	Constants of Free Space	674
B-2	Physical Constants of Electron and Proton	674

Equation (6-23) enables us to find the vector magnetic potential  $A$  from the volume current density  $J$ . The magnetic flux density  $B$  can then be obtained from  $\nabla \times A$  by differentiation, in a way similar to that of obtaining the static electric field  $E$  from  $-\nabla V$ .

Vector potential  $A$  relates to the magnetic flux  $\Phi$  through a given area  $S$  that is bounded by contour  $C$  in a simple way:

$$\Phi = \int_S B \cdot ds \quad (6-24)$$

The SI unit for magnetic flux is weber (Wb), which is equivalent to tesla-square meter ( $T \cdot m^2$ ). Using Eq. (6-15) and Stokes's theorem, we have

$$\Phi = \int_S (\nabla \times A) \cdot ds = \oint_C A \cdot d\ell \quad (\text{Wb}). \quad (6-25)$$

Thus, vector magnetic potential  $A$  does have physical significance in that its line integral around any closed path equals the total magnetic flux passing through the area enclosed by the path.

## 6-4 The Biot-Savart Law and Applications

In many applications we are interested in determining the magnetic field due to a current-carrying circuit. For a thin wire with cross-sectional area  $S$ ,  $dv$  equals  $S d\ell$ , and the current flow is entirely along the wire. We have

$$J dv = JS d\ell = I d\ell, \quad (6-26)$$

and Eq. (6-23) becomes

$$A = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\ell'}{R} \quad (\text{Wb/m}), \quad (6-27)$$

where a circle has been put on the integral sign because the current  $I$  must flow in a closed path,<sup>†</sup> which is designated  $C$ . The magnetic flux density is then

$$\begin{aligned} B &= \nabla \times A = \nabla \times \left[ \frac{\mu_0 I}{4\pi} \oint_C \frac{d\ell'}{R} \right] \\ &= \frac{\mu_0 I}{4\pi} \oint_C \nabla \times \left( \frac{d\ell'}{R} \right). \end{aligned} \quad (6-28)$$

<sup>†</sup> We are now dealing with direct (non-time-varying) currents that give rise to steady magnetic fields. Circuits containing time-varying sources may send time-varying currents along an open wire and deposit charges at its ends. Antennas are examples.

## 6-4 The Biot-Savart Law and Applications

235

It is very important to note in Eq. (6-28) that the *unprimed* curl operation implies differentiations with respect to the space coordinates of the *field point*, and that the integral operation is with respect to the *primed source coordinates*. The integrand in Eq. (6-28) can be expanded into two terms by using the following identity (see Problem P.2-37):

$$\nabla \times (f\mathbf{G}) = f\nabla \times \mathbf{G} + (\nabla f) \times \mathbf{G}. \quad (6-29)$$

We have, with  $f = 1/R$  and  $\mathbf{G} = d\mathbf{e}'$ ,

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \oint_C \left[ \frac{1}{R} \nabla \times d\mathbf{e}' + \left( \nabla \frac{1}{R} \right) \times d\mathbf{e}' \right]. \quad (6-30)$$

Now, since the unprimed and primed coordinates are independent,  $\nabla \times d\mathbf{e}'$  equals 0, and the first term on the right side of Eq. (6-30) vanishes. The distance  $R$  is measured from  $d\mathbf{e}'$  at  $(x', y', z')$  to the field point at  $(x, y, z)$ . Thus we have

$$\begin{aligned} \frac{1}{R} &= [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2}; \\ \nabla \left( \frac{1}{R} \right) &= \mathbf{a}_x \frac{\partial}{\partial x} \left( \frac{1}{R} \right) + \mathbf{a}_y \frac{\partial}{\partial y} \left( \frac{1}{R} \right) + \mathbf{a}_z \frac{\partial}{\partial z} \left( \frac{1}{R} \right) \\ &= -\frac{\mathbf{a}_x(x-x') + \mathbf{a}_y(y-y') + \mathbf{a}_z(z-z')}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}} \\ &= -\frac{\mathbf{R}}{R^3} = -\mathbf{a}_R \frac{1}{R^2}, \end{aligned} \quad (6-31)$$

where  $\mathbf{a}_R$  is the unit vector directed from the source point to the field point. Substituting Eq. (6-31) in Eq. (6-30), we get

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\mathbf{e}' \times \mathbf{a}_R}{R^2} \quad (1). \quad (6-32)$$

Equation (6-32) is known as *Biot-Savart law*. It is a formula for determining  $\mathbf{B}$  caused by a current  $I$  in a closed path  $C'$  and is obtained by taking the curl of  $\mathbf{A}$  in Eq. (6-27). Sometimes it is convenient to write Eq. (6-32) in two steps:

$$\mathbf{B} = \oint_C d\mathbf{B} \quad (1), \quad (6-33a)$$

with

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \left( \frac{d\mathbf{e}' \times \mathbf{a}_R}{R^2} \right) \quad (1), \quad (6-33b)$$

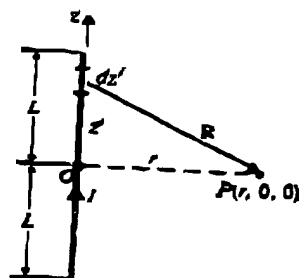


FIGURE 6-5

A current-carrying straight wire (Example 6-4).

which is the magnetic flux density due to a current element  $I dz'$ . An alternative and sometimes more convenient form for Eq. (6-33b) is

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \left( \frac{d\mathbf{e}' \times \mathbf{R}}{R^3} \right) \quad (7)$$

(6-33c)

Comparison of Eq. (6-32) with Eq. (6-10) will reveal that Biot-Savart law is, in general, more difficult to apply than Ampère's circuital law. However, Ampère's circuital law is not useful for determining  $\mathbf{B}$  from  $I$  in a circuit if a closed path cannot be found over which  $\mathbf{B}$  has a constant magnitude.

**EXAMPLE 6-4** A direct current  $I$  flows in a straight wire of length  $2L$ . Find the magnetic flux density  $\mathbf{B}$  at a point located at a distance  $r$  from the wire in the bisecting plane (a) by determining the vector magnetic potential  $\mathbf{A}$  first, and (b) by applying Biot-Savart law.

**Solution** Currents exist only in closed circuits. Hence the wire in the present problem must be a part of a current-carrying loop with several straight sides. Since we do not know the rest of the circuit, Ampère's circuital law cannot be used to advantage. Refer to Fig. 6-5. The current-carrying line segment is aligned with the  $z$ -axis. A typical element on the wire is

$$d\mathbf{e}' = \mathbf{a}_z dz'$$

The cylindrical coordinates of the field point  $P$  are  $(r, 0, 0)$ .

a) By finding  $\mathbf{B}$  from  $\nabla \times \mathbf{A}$ . Substituting  $R = \sqrt{z'^2 + r^2}$  into Eq. (6-27), we have

$$\begin{aligned} \mathbf{A} &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \int_{-L}^L \frac{dz'}{\sqrt{z'^2 + r^2}} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \left[ \ln(z' + \sqrt{z'^2 + r^2}) \right]_{-L}^L \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L} \end{aligned} \quad (6-34)$$

**KIRJALLISUUS**

- [1] Rautio, K., Tunturi T-road juoksumaton ohjelmistosuunnitelma. Mariachi Oy, Turku, 2000.
- [2] Peltonen, P., Toteutusmäärittely TIE käyttöliittymäelektronikka (IUE). Mariachi Oy, Turku, 1998.
- [3] Lamberg, P., T-road schema. Mariachi Oy, Turku, 2000.
- [4] Rautio, K., Tunturi T-road juoksumatto ala- ja yläkortin välinen viestintä. Mariachi Oy, Turku, 2000.
- [5] Rautio, K., TIE IUE ohjelmiston toteutus. Mariachi Oy, Turku, 1998.
- [6] Edwards, S., The Heart Rate Monitor Book. Fleet Feet Press, Sacramento, 1993.
- [7] Polar OEM handbook Version 1.3. Polar Electro Oy, Kempele, 2000.
- [8] Polar Electro, Operational and technical description T31 transmitter. Polar Electro Oy, Kempele, 2000.
- [9] Polar Electro, Operational and technical description T41 transmitter. Polar Electro Oy, Kempele, 2000.
- [10] Polar Electro, PCBA receiver operational and technical description. Polar Electro Oy, Kempele, 2000.
- [11] Cheng, D., Field and Wave Electromagnetics. Addison-Wesley Publishing Company, New York, 1991.
- [12] Voipio, E., Sähkö- ja magneettikentät. Otakustantamo, Helsinki, 1976.
- [13] Millman, J. ja Grabel, A., Microelectronics, 2<sup>nd</sup> edition. McGraw-Hill Book Co, Singapore, 1987.
- [14] <http://pdf.toshiba.com/tasc/components/Datasheet/C807.pdf>  
www-dokumentti, luettu 10.7.2001.